Chapter 2

Stacking

I've often marvelled at the skill of architects who design high-rise buildings. I'm not saying I find them attractive (the buildings that is) apart from maybe the Empire State Building with its classic art deco styling. When I was a kid it held legendary status as the tallest building in the world, but I've since come to realise they cheated by parking *Thunderbird 1* on top!

In a sense a skyscraper is just a pile of identical boxes, so what's the big deal in that? True, but there are other architectural issues that present themselves which are unique to them. The one that has always fascinated me is the plumbing. How *do* they design and incorporate (in such a way that no-one pays the slightest attention) a plumbing system that overcomes all the problems of gravity and pressure? Anyway, fascinating though plumbing is, I still don't know. It's actually the pile of boxes I'm interested in because I want us to look at a practical way to consider the most essential difference between one dimension and the next.

There is a basic symmetry that runs through all of nature and it may be seen virtually everywhere, like a blueprint for how the world works. Consider this: quarks and leptons make atoms, atoms make molecules, molecules make minerals, minerals make sand, sand makes bricks, bricks make walls, walls make buildings, buildings make cities and cities make the world.

As I sit here with my little computer I can see that myriads of tiny black pixels combine to form letters and these combine to form words, all the words together constitute the document which assumes its place inside a folder – and there are loads of folders on my computer. Pixels, letters, words, documents, folders, computer.

Or again, consider the humble thread… thread weaves a garment and garments make a collection, a collection snowballs a trend, and a trend clothes the world. The examples are endless. Think about the way biologists categorise animals and plants; or how your own chemistry forms organelles within cells, forming tissues, organs and limbs, and the whole lot is stored in the giant skin-folder you call your body, which is part of a family, community, state, nation, the world! Indeed, not only does the world exist all around us in this form, but the principle is also reflected in the stars and galaxies of space as UK Astronomer Royal, Sir Martin Rees tells us, *'Cosmic structure forms hierarchically, from the bottom up.'*[a]

So what we are doing?

We are leaping in easy, manageable stages from the micro to the macro by grouping things together. We are 'stacking up' each completed group till it forms the next. The thing to notice is that each new level is complete, but is somehow greater than the sum of its parts, forming a new entity with a unique identity or function that might not have been able to be predicted from knowledge of its parts alone. Oxford Professor of Physics, Frank Close describes how such groupings are critical to our investigation of nature,

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[a] Martin Rees, *Just Six Numbers*, Phoenix 2001, P121

> *'The reason that predictive science is possible, even though the fundamental equations may be unknown or, if known, be impossible to solve, is because it is not just atoms and molecules that respect organization: laws that operate at the level of individual atoms become organized into new laws as one moves up to complex systems.'* [a]

He continues,

> *'It is this hierarchy of structures and laws that enables us to understand and describe the world; the outer layers rely on the inner yet they each have an identity and can often be treated in isolation.'* [b]

The mid-20th Century writer Arthur Koestler suggested using the Greek word *holon* to mean a whole that is made up of parts which are each in turn wholes, however (I suppose with it being a fairly silly-sounding word in English) it didn't really catch on. But the very fact that he felt the need to invent one in the first place shows the extent to which this essential characteristic of reality is perhaps somewhat taken for granted. The correct term for the phenomenon is *nested hierarchies*. Mathematically they are sets, with each able to contain a virtually infinite number of the set before, and they belong in the realm of complexity theory. However, in this book we shall take a *Flatland*-based geometrical approach, not least for its powerful visuals and not-to-be-sneered-at simplicity, because, as mathematician Ian Stewart tells us in his book *Flatterland*, *'something which seems very complicated may in fact arise from simple rules.'*[c]

The fact that nested hierarchies are so deeply ingrained in nature, pervading not just our organisational thinking but the way the whole of reality appears to be ordered, suggests to me that they must represent structure at its most fundamental and profound. It is my belief that the original blueprint of creation lies mathematically and philosophically exposed to our view in the simple and enduring geometry of Edwin Abbott Abbott's *Flatland*.

The Basics

Take a dot, draw another dot next to it, then add to it a large number of points one after the other What do you get? You get a line.

Fig.2 Points stack to form a line.

If you do the same with the line and lay more lines out alongside it, the lines become a plane (a flat surface).

Fig.3 Lines stack to form a plane.

[a] Frank Close, *Nothing: A Very Short Introduction*, Oxford 2009, P92
[b] Ibid., P93
[c] Ian Stewart, *Flatterland*, Pan Books 2003, P87

We have moved from a

- 0-Dimensional entity, which is a point, up to a
- 1-Dimensional entity, which is a line (having length only), then up to a
- 2-Dimensional entity, which is a plane (having length *and* width) – and all by simply adding *more of the same* each time, side by side.

What we are doing is leaping from one dimension to the next, ***forming the next dimension by compressing together a large number of the last***. Whether the required number is infinite or not is unclear, and not really important, because once combined they lose their individual identity and 'fuse' together. The main thing is that the principle holds good.

Now, continuing the process… if we stack up the plane like stacking sheets of paper on top of one other, eventually we get a 3D block like a ream of paper, possessing length, width *and* height. Hey presto, a solid object! And we started out with nothing.

Or at least a 0-Dimensional entity.

Meet the Principle of Stacking…

In the next chapter we will make a brief list of simple principles inherent in the way that *Sphere* manifested his cross-sectional presence to unsuspecting *A Square* of Flatland. In the meantime we see, not only that *Sphere* may be split down into cross-sections, but that those very same 2D slices are what his 3D higher dimension is actually constructed out of. If we remember, this is how *Sphere* himself put it as he attempted to enlighten *A Square*,

> *"You call me a Circle; but in reality I am not a Circle, but an infinite number of Circles, of size varying from a Point to a Circle of thirteen inches in diameter, one placed on the top of the other."*

Geometrically, a sphere is composed of an indefinitely high number of circles, all stacked one on top of another. The circles (disks) grow and grow in size until, reaching the sphere's 'equator', they then shrink down until they disappear. There is a very powerful principle at work here. Let's call it…

The Principle of Stacking:

Each dimension is composed of an indefinitely high number of cross-sections (slices) of the dimension below, stacked together and fused into a single entity.

Beauty and consistency begin to emerge, and stacking cannot be underestimated as a powerful, dimensionally descriptive tool. Here's US physicist Rudy Rucker on the subject of stacking,

> *'A sphere is a three-dimensional stack of circles;'*

But Rucker goes on to lift us up by another dimension into the fourth when he says that,

> *'a hypersphere is a four-dimensional stack of spheres.'* [a]

A stack of spheres? How do you stack spheres?

Unfortunately no-one can actually visualise a 4D hypersphere because, for some reason, we humans are limited to only being able to see and imagine in 3D. In theory there is no good reason why a

[a] Rudy Rucker, *The Fourth Dimension*, Houghton Mifflin Company 1984, P19. A book written to commemorate the Centenary Year of *Flatland*.

hypersphere can't exist, but it is difficult for us to comprehend what a *'stack of spheres'* might actually look like. We might imagine a thousand footballs on top of one another, or a kiddies' multi-coloured ball pool – but we are wasting our time because the thing we see is still 3-Dimensional due to the gaps between each ball, and stacking does *not* permit gaps. But it is enough for us to notice, at least theoretically (i.e. mathematically), that dimensional stacking goes on and on, through 4D to 5D to 6D...

We will come to hyperspheres in due course, but for now let's just focus on what we *can* actually imagine, because we may learn so much from just the simple idea of a sphere passing through a plane. Our friends *Sphere* and *A Square* will crop up frequently throughout this book, because simplicity is the key, and they tell us all we need to know. We will extrapolate their simple geometrical relationship of stacking and cross-sections right up and down the dimensional structure, wielding it like a magical key on our quest to unlock the structure's many secrets.

Oops Upside Your Head

But there's something else we need to notice: a higher dimensional entity such as *Sphere* may appear to a lower in slices 'cut' from any angle and is not constrained to some fixed stacking arrangement. Why? Because they have all fused together. Even though *Sphere's* slices are 2D as they pass through Flatland, the stacked entity that is *Sphere himself* is 3D, and therefore what he is made of obeys 3-Dimensional laws. *Sphere* may therefore be considered stacked up from any conceivable angle because, although a 3D object is theoretically built up from 2D cross-sectional 'slices', ***once they have fused together it is not possible to discern any particular stacking arrangement***.

This is why in our own world there is no way to tell which direction in space is length, which direction is width and which is height. They are all obviously there, but our 3-Dimensionality dictates that there are infinite possibilities based on three possible stacking options[a] (which mathematicians call degrees of freedom). Clearly, all this tells us nothing of the order in which *Sphere* became stacked, only that any such order becomes indiscernible once a dimension is complete, which takes us right back to nested hierarchies. All the 'joins' disappear so that, just as digital becomes analogue, the discrete becomes smooth. We find in this a principle which is of supreme importance to science. Italian theoretical physicist, Carlo Rovelli writes that,

> *'Einstein discovered that the Newtonian space described by geometry is in fact a field, like the electromagnetic field, and fields are nicely continuous and smooth only if measured at large scales. In reality, they're quantum entities that are discrete and fluctuating.'* [b]

Although Rovelli believes that this phenomenon could spell the end of geometry as a useful description of physical space, we see from the simplicity of *Flatland* interactions how both are able to function in tandem[c]. At least in this instance, the simple principles of *Flatland* mean we can have our cake and eat it. This is what gives each new dimension its own new and unique character – a character which one would *not* have been able to predict from its cross-sections because the cross-section is always of the dimension below. For ease of reference, we will enshrine this concept within a second dimensional principle:

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[a] A 1D entity (a line) has only one stacking option: the 0-Dimensional points must be aligned end on end along the line. Stacking options run according to the number of dimensions: one stacking option for a 1D line. Think about 2D flatness and you will see it has two directions in which lines could be stacked.

[b] Carlo Rovelli, *Geometry*, from *This Idea Must Die*, Edited by John Brockman, Harper Perennial 2015, P473

[c] As do quantum and classical physics!

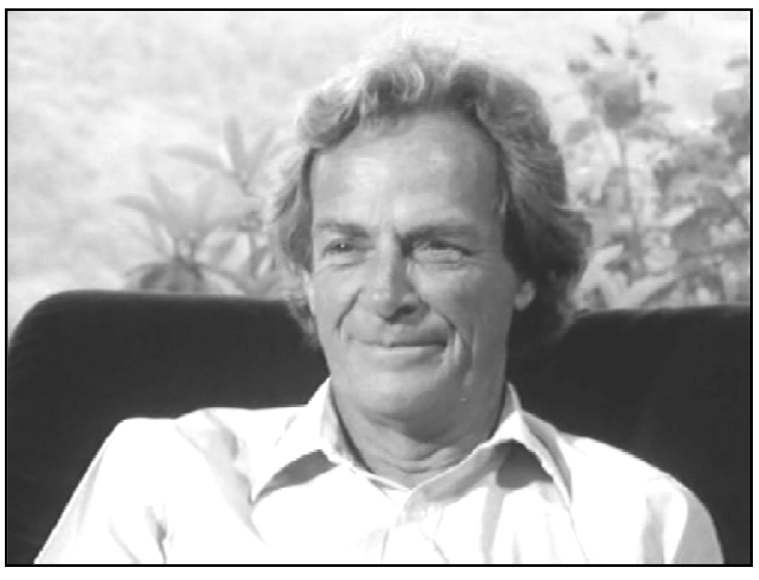
The Principle of Character:

Once the stacking of a dimension is complete it assumes a whole new character. Its individual cross-sections fuse together and their discrete nature becomes indiscernible.

Uxles and Wuxles

This principle, along with our first, will re-appear frequently as this book unfolds[a]. And because their roots lie in geometry, they offer us a sword of reason with which we may slay the dragon of 'new age' disinformation that plagues the whole subject of dimensions – a dragon which (as is the way with dragons) may be barring our way to untold riches!

Interestingly, the *Principle of Stacking*[b] is a bit of a one-way street, describing a process which acts in one direction only. Have a listen to eminent US physicist, the late, great Richard P Feynman as he describes the enduring philosophical mystery of *causality* in his own inimitable style,



"Now if the world of nature is made of atoms, and we too are made of atoms and obey physical laws, the most obvious interpretation of this evident distinction between past and future, and this irreversibility of all phenomena, would be that some laws, some of the motion laws of the atoms, are going one way – that the atom laws are not such that they can go either way. There should be somewhere in the works some kind of principle that uxles only make wuxles, and never vice versa, and so the world is turning away from uxley character to wuxley character all the time – and this one-way business of the interactions of things should be the thing that makes the whole phenomena of the world seem to go one way.
But we have not found this yet." [c]

Fifty years have gone by since Feynman's lecture with no discernible progress, and the 'arrow of time' remains a scientific enigma. However, as Feynman says, "*There should be somewhere in the works some kind of principle…*"

The whole world – indeed the entire universe – is awash with stacking, and the individual character of virtually anything you can think of is the product of stacking at some more fundamental level, because the *Principle of Stacking* requires that each dimension begins at one and multiplies, tending toward completion, before repeating the process with (necessarily) the next dimension. In this way, stacking turns away *'from uxley character to wuxley character all the time'*. A stacked and 'nested' dimension is indivisible as our experience of 3D shows, therefore it seems to me that a principle may indeed exist which describes *'this one-way business of the interactions of things'*, but perhaps it is just so commonplace that we take it for granted.

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[a] Every time one of our *Flatland*-derived principles is alluded to, it will appear in full in a footnote.
[b] *The Principle of Stacking:* Each dimension is composed of an indefinitely high number of cross-sections (slices) of the dimension below, stacked together and fused into a single entity.
[c] Richard Feynman, *The Character of Physical Law*, Penguin Press Science 1992

Although all this may seem fairly ordinary at the level of the 1st, 2nd and 3rd Dimensions, the *Principle of Stacking* may hold serious and very exciting implications. Using the (Occam's) razor edge of its very simplicity, we are about to systematically dissect reality.